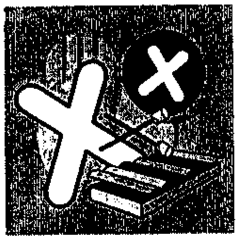
Chapter 5: Factoring



Notes
10

5.1: Factoring by GCF and Grouping

5.2: Factoring Trinomials Form $x^2 + bx + c$

5.3: Factoring Trinomials Form $ax^2 + bx + c$

5.4: Factoring Difference of Two Squares and Factor Perfect Square Trinomials

5.5: Factoring Using Multiple Methods

5.6: Solve Quadratic Equations by Factoring

5.7: Simplify Rational Expressions

5.8: Factoring Applications

5.1: Factoring by GCF and Grouping

What is factoring?

Find the prime factors of:

6

$\boxed{3 \cdot 2}$

27

3 9

3 3·3

250

25 10

5·5 2 5

$25x^2y^5$

5 5 x x y y y y y

$100ab^3$

10 10

5 2 5 2 a b b b

$\boxed{5 2 \cdot 5 2 a b b b}$

$45x^4y^2z$

9 5

9 5 x x x x y y z

The greatest common factor (GCF)- the largest common factor of the integers.

To Find the GCF:

1. Find the prime factorization of the terms
2. Find the common factors in each of the terms and the most of each in all terms
3. Multiply the most common factors in all of the terms.

Example: Find the GCF of: 36 and 90

36	90
$6 \cdot 6$	$9 \cdot 10$
$2 \cdot 3 \cdot 2 \cdot 3$	$3 \cdot 3 \cdot 2 \cdot 5$
$2 \cdot 2 \cdot 3 \cdot 3$	$2 \cdot 3 \cdot 3 \cdot 5$

The common factors are: 2, 3

The most 2's in common is: 2
The most 3's in common is: $3 \cdot 3$

The GCF is: $2 \cdot 3 \cdot 3 = 18$

Practice Example: Find the GCF of: 24 and 70

24	70
4 6	7 10
$2 \cdot 2 \cdot 2 \cdot 3$	$7 \cdot 5 \cdot 2$

GCF: 2

Practice Example:
Find the GCF of: 45 60 75

45	60	75
9 5	6 10	25 3
$3 \cdot 3 \cdot 5$	$3 \cdot 2 \; 5 \cdot 2$	$5 \cdot 5 \cdot 3$

GCF $3 \cdot 5 = 15$

Example: Find the GCF of: $9x^3y^2$ and $15xy^4$

$9x^3y^2$ $15xy^4$

$3 \cdot 3xxxyy$ $3 \cdot 5xyyyy$

The common factors are: 3,x, y

The most 3's in common is: 3
The most x's in common is: x
The most y's in common is: yy

The GCF is: $3xyy = 3xy^2$

Practice Example: Find the GCF of: $20x^5y^6$ and $150x^7y^3$

4 5
$2 \cdot 2 \cdot 5 x^5 y^6$

10 15
$2 \cdot 5 \quad 3 \cdot 5 \ x^7 y^3$

$2 \cdot 5 \quad x^5 y^3$

$10x^5y^6$

Practice Example:
Find the GCF of: $42a^3b^2$ $63ab^5$ $21a^5b^4$

7 6
$7 \cdot 6 \cdot a^3b^2$

7 9
$7 \cdot 3 \cdot 3$

7 3
$7 \cdot 3$

$3 \cdot 7$

21 GCF
ab^2

Simplify: $3x(2x+5)$

$6x^2+15x$

Now work backwards to get the original back

$3x(2x+5)$

Factor using Greatest Common Factor

Example: Factor- $25x^3y^3+15x^5y^6-5x^2y^7$

If we break down the polynomial into prime factors we would have:

$5\cdot 5xxxyyy + 5\cdot 3xxxxxyyyyyy - 5xxyyyyyyy$

What are the most factors you can pull out of all of the terms?

$5x^2y^3$

Then what remains once that is pulled out?
Divide by what you are factoring out to see what remains.

$$\frac{25x^3y^3}{5x^2y^3}+\frac{15x^5y^6}{5x^2y^3}-\frac{5x^2y^7}{5x^2y^3} =$$

$5x \quad + 3x^3y^3 \quad - y^4$

The factored form is: $5x^2y^3(5x-3x^3y^3-y^4)$

You can check by simplifying.

Practice Examples:

Factor:

$10a^2b^4 + 15a^3b^2$

a^2b^2

$5a^2b^2(2b^2 + 3a)$

Factor:

$3x^5y^{10} - 9x^7y^4 + 21x^2y^{12}$

$3x^2y^4(x^3y^6 - 3x^5 + 7y^8)$

Factor:

$65y^9v^{18} + 20y^{30}v^{20} + 30y^{18}v^4$

$5y^9v^4(13y^{14} + 4y^{21}v^6 + 6y^4)$

Factor using Grouping

Factor:
$3(x+2)-x(x+2)$

3b − xb

b(3 − x)

(x + 2)(3 − x)

Example: Factor- $2ys-2yf+bs-bf$

Method 1 will not work because you can not pull out of all of the terms. Since there are 4 terms, we can group 2 in common and another 2 in common.

$2ys-2yf$ and $bs-bf$

2ys − 2yb + bs − bf

Now, we can pull out from each group:

2y(s − f) b(s − f)

$2y(s-f)$ and $b(s-f)$

If this is factorable, then what is the parenthesis will match. Pull out what they both have in common and see what remains.

$(s-f)(2y+b)$, this is the factored form.

You can check by simplifying.

Practice Example:

Factor:
$xy+3y-6x-18$

y(x + 3) − 6(x + 3)

(x + 3)(y − 6)

xy − 6x + 3y − 18

5.1: Factoring by GCF and Grouping Practice Problems Continue

9. Factor: $x(x+3)+4(x+3)$

$x^2+3x+4x+12$

$(x+3)(x+4)$

10. Factor: $x^2(2x-7)-3x(2x-7)+2(2x-7)$

$2x^2-7x-6x-21x+4x-14$

$(2x-7)(x^2-3x+2)$

11. Factor: $ax+ay+bx+by$

$ax+ay$ $bx+by$

$a(x+y)+b(x+y)$

$(x+y)(a+b)$

12. Factor: $x^2+x-2x-2$

x^2+x $-2x-2$

$x(x+1)$ $-2(x+1)$

$(x+1)(x-2)$

13. Factor: $xy-xz-ay+az$

$x(y-z)-a(y-z)$

$(y-z)(x-a)$

14. Factor: $15x^2-10xy+6x-4y$

$15x^2-10xy$ $+6x-4y$

$5x(3x-2y)+2(3x-2y)$

$(3x-2y)(5x+2)$

15. Factor: $2a^2-3ab-2a+3b$

$2a^2-3ab$ $-2a+3b$

$2a(a-1)$ $-3b(b-1)$

$(a-1)(2a-3b)$

16. Factor: $20x^2+10xy-16x-8y$

$2(x^2+10xy-16x-8y)$

$10x(2x+5y)-8(2x+5y)$

$(2x+5y)(10x-8)$

5.2: Factoring Trinomials Form x^2+bx+c

How do you factor trinomials in the form x^2+bx+c?

Simplify: $(x+3)(x+4)$

Solution:

$(x+3)(x+4)$

$F \quad O \quad I \quad L$

$x \cdot x \; +4x \; +3x \; +3 \cdot 4$

$x^2+4x+3x+12$

$F \quad O+I \quad L$

$x^2 \quad +7x \quad +12$

Notice to get the first term, it is the **multiplication** of the **F** part of FOIL

Notice to get the inside term, it is the **addition** of the **O + I** of FOIL

Notice to get the outside term, it is the **multiplication** of **L** of FOIL.

Simplify: $(x-2)(x+5)$

Example: Factor- $x^2 + 7x + 12$

Pulling out the GCF or Grouping will not work because you can not pull out of all of the terms and there is not an even number of terms to group.

Our goal is to factor $x^2 + 7x + 12$ into two factors that are being multiplied: $(\quad)(\quad)$

To get the x^2, the factors must be $x \cdot x$ in the front (F of FOIL).

To factor this polynomial where the leading coefficient is 1:

We need to two numbers that will multiply to the last term of: + 12 (L of FOIL) and add to the middle term of: + 7 (O + I of FOIL)

We look at the factors of +12 and find: $1 \cdot 12$, $2 \cdot 6$, $3 \cdot 4$

$-1 \cdot -12$, $-2 \cdot -6$, $-3 \cdot -4$

Which pair will multiply to +12 and add to +7?

The numbers are: +3 and +4. $(3)(4) = 12$ $3 + 4 = 7$

******Make sure your signs are correct*****

The factored form is: $(x+3)(x+4)$

You can check by simplifying.

$(x+3)(x+4)$

$x \cdot x + 4x + 3x + 3 \cdot 4$

$x^2 + 4x + 3x + 12$

$x^2 + 7x + 12$

Know Your Signs

Multiply: +

Then the signs are:

\+ + **OR** - -

Add: +

\+ +

Add: -

\- -

Multiply: -

Then the signs are:

\+ -

5.2: Factoring Trinomials Form x^2+bx+c Practice Examples Continue

9. Factor: x^2-2x+1

$(x-1)(x-1)$

$(x-1)^2$

10. Factor: $35+x^2-12x$

$x^2-12x-35$

$(x-7)(x-5)$

11. Factor: $x^2+23x-50$

$(x-2)(x+2)$

12. Factor: $x^2+7x-12$

Prim

No solution

13. Factor: $x^2+27x+72$

$(x-2)(x+3)$

14. Factor: $x^2-9x-36$

$(x+3)(x-12)$

6, 6

9, 4

12, 3

15. Factor: $-x^2-7x+8$

$(x-8)(x-1)$

16. Factor: $3x^2-3x-18$

6, 3

2, 9

$(3x+3)(x-6)$

$x^2-6+3x-18$

$3x^2-3x-18$

5.3: Factoring Trinomials Form $ax^2 + bx + c$

How do you factor trinomials in the form $ax^2 + bx + c$?

Example: Factor- $12x^2 - 23x + 5$

Method 1 or Method 2 or Method 3 will not work because:

You can not pull out of all of the terms (Method 1 Pull out GCF),

There is not an even number of terms to group (Method 2 Grouping), and

Since the leading coefficient is not 1, (Method 3 Trinomial a = 1)

You still need two numbers that multiply to the last term, but they will NOT add to the middle term coefficient because the leading coefficient is not 1.

There are two ways to factor this polynomial: Trial and Error and Change to grouping

First Method: Trial and Error

We know that when we factor $12x^2 - 23x + 5$, the factored form will be: $(\quad)(\quad)$

Step 1: We will first list the possible factors:

We must get the leading term of $12x^2$

The possibilities are: $(1x)(12x)$ $(2x)(6x)$ $(3x)(4x)$

Step 2: We know that it must multiply to the last term of $+5$:

The possibilities with signs are: $(+1)(+5)$ $(-1)(-5)$ ****Signs are VERY important****

Since the middle term is negative, and it must add to the middle term, then it must be: $(-1)(-5)$

Step 3: Then try different combinations and simply each one to try to get: $12x^2 - 23x + 5$

Ex. $(2x-5)(6x-1)$ $(x-5)(12x-1)$ $(3x-1)(4x-5)$ $(3x-5)(4x-1)$

$(3x-5)(4x-1)$, this is the factored form. You can check by simplifying.

Second Method: Change to grouping

Example: Factor- $12x^2 - 23x + 5$

In this method we want the trinomial to become a polynomial with four terms so we can perform grouping. There is a specific way to change the trinomial into a polynomial with four terms.

Step 1: Find the middle factors

Multiply the leading coefficient by the last constant term.

$12 \cdot 5 = +60$, we want to find factors that multiply to get this number (including the sign)

The factors must also add to the middle term coefficient, in this case -23.

The factors that multiply to +60 and add to -23 are: -20 and -3

Step 2: Rewrite the polynomial

We found the two coefficients to replace the middle term to form a polynomial with 4 terms.

The trinomial $12x^2 - 23x + 5$ will now become:

$$12x^2 - 3x - 20x + 5$$

It does not matter which number goes first, but since we are going to group, you want to place factors next to terms they have something in common with.

For example, place the "-3x" next to the "$12x^2$" because they have a "3x" in common and place the "-20x" next to the "5" because they have a "5" in common.

Step 3: Use Grouping

$$12x^2 - 3x - 20x + 5$$

$$12x^2 - 3x \qquad -20x + 5$$

$$3x(4x-1) \qquad -5(4x-1)$$

$$(4x-1)(3x-5)$$

$(4x-1)(3x-5)$, this is the factored form. You can check by simplifying.

Practice Example:

Factor:

$2x^2 + 5x + 3$

$2x^2 + 2x + 3x + 3$

Factor:

$9x^2 - 9x + 2$ $\quad -9 \quad -6 \quad -3$

$9x^2 - 6x \quad -3x + 2$

$3x(3x-2) \quad -1(3x-2)$

$(3x-2)(3x-1)$

$9x^2 - 3x \quad -6x + 2$

$3x(3x-1) \quad -2(3x-1)$

Factor:

$10x^2 + 13x - 3$

$10x^2 - 2x + 15x - 3 \qquad -2 + 15$

$2x(5x-1)$

Factor:

$3x^2 - 7x - 6$ $\quad -18$

$3x^2 - 9x + 2x - 6$

$3x^2 - 9x \qquad 2x - 6$

$3x(x-3) \qquad 2(x-3)$

$(x-3)\ (3x+2)$

	x	-3
$3x$	$3x^2$	$-9x$
$+2$	$+2x$	-6

Factor:

$-8x^3 + 2x^2 + 3x$

$-x\,(8x^2 - 2x - 3)$ -2 + -2 add

$8x^2 + 4x - 6x - 3$

$8x^2 + 4x \quad -6x - 3$

$4x(2x+1) \quad -3(2x+1)$

$(2x+1) \quad (4x-3)$

$x\,(2x+1)(4x-3)$

$-x\,(2x+1)(4x-3)$

$(-2x^2 - x)(4x-3)$

$-8x^3 + 6x^2 - 4x^2 + 3x$

$-8x^3 + 2x^2 + 3x$

Factor:

$12x^5 - 17x^4 + 6x^3$

$x^3(12x^2 - 17x + 6)$

12 mult

17 add

$12x^2 \; 9x \; -8x \; 6$

-8

$12x^2 - 9x \quad -8x + 6$

$3x(4x-3) \quad -2(4x-3)$

$(4x-3)(3x-2)$

$x^3(4x-3)(3x-2)$

$x^3(4x-3)(3x-2)$

$(4x^4 - 3x^3)(3x-2)$

$12x^5 - 8x^4 - 9x^4 + 6x^3$

$12x^5 - 17x^4 + 6x^3$

How do you factor trinomials in the form $ax^2 + bx + c$?

follow the method

5.3: Factoring Trinomials Form $ax^2 + bx + c$ Practice Examples

1. Factor: $2x^2 + 7x + 3$

$2x^2 + 7x + 3$ +6 mult
+7 add

$2x^2 \quad +6x \quad +1x \quad 3$

$2x^2 + 6x \qquad 1x + 3$

$2x(x+3) \qquad 1(x+3)$

$(x+3)(2x+1)$

2. Factor: $6x^2 + 7x + 2$

$6x^2 + 7x + 2$

$6x^2 + 3x \quad +4x \quad +2$

$6x^2 + 3x \qquad 4x + 2$

$3x(2x+1) \qquad 2(2x+1)$

$(2x+1)(3x+2)$

3. Factor: $8x^2 + 10x - 3$ −24 mult
+10 add

$8x^2 - 2x \quad +12x \quad -3$

$2x(4x-1) + 3(4x-1)$

$(4x-1)(2x+3)$

4. Factor: $6x^2 - x - 12$ [illegible]
−1 add

$6x^2 - x - 12$

$6x^2 \quad -9x \quad +8x \quad -12$

$3x(2x-3) \qquad 4(2x-3)$

$(2x-3)(3x+4)$

5. Factor: $4x^2 + 4x - 15$ −60 mult
+4 add

$4x^2 \quad -10x \quad +6x \quad -15$

6 10

$2x(2x-5) \qquad 3(2x-5)$

$(2x-5)(2x+3)$

6. Factor: $5x^2 - 17x + 6$ +30 mult
−17 add

$5x^2 - 17x + 6$

$5x^2 \quad -15x \quad -2x \quad +6$ −15 −2

$5x(x-3) \qquad -2(x-3)$

$(x-3)(5x-2)$

5.3: Factoring Trinomials Form ax^2+bx+c Practice Examples Continue

7. Factor: $25x^2+21x-4$

-100 mult
+21 add

$25x^2+21x-4$

$25x^2+25x-4x-4$

$25(x+1)-4(x+1)$

$(x+1)(25x-4)$

8. Factor: $12x^2-25x+12$

144 mult
-25 add

$12x^2 -16x -9x +12$

$-16 \cdot -9$

$4x(3x-4) -3(3x-4)$

$(3x-4)(4x-3)$

9. Factor: $20x^2-31x-7$

140 mult
-31
-35 +4

10. Factor: $2x^2+3x+9$

Prime

11. Factor: $2x^3+4x^2-30x$

$2x(x^2+2x-15)$

$2x(x+5)(x-3)$

12. Factor: $-3x^2+9x+54$

$-3(x^2-3x-18)$

$-3(x-6)(x+3)$

$(-3x+18)(x+3)$

1) Pull out the GCF
2) Grouping (4 terms)
3) Trinomial (guess)
Trinomial ... $3x^2+7x+6$
Special case (+/-) ex $4x^2-9$ (two term)

5.4: Factoring Difference of Two Squares and Factor Perfect Square Trinomials

How do you factor difference of two squares?

Binomial (+/-) Special Case (Difference of Two Squares)

Example: Factor- $4x^2 - 9$

Method 1 or Method 2 or Method 3 or Method 4 will not work because you can not pull out of all of the terms, there is not an even number of terms to group, and it is not a trinomial.

This is a Special Case because:

$4x^2 - 9$
$(2x+3)(2x-3)$
$4x^2 - 6x + 6x - 9$

1) There are only 2 terms
2) There is a (-) in the middle
3) You can take the square root of both terms

If all the criteria above represented, then you have **difference of squares**.

To factor this polynomial, you take the square root of both terms:

2x and 3

Since there is no middle term, it must have canceled each other out.

Therefore, you make one factor with (+) and another factor with (-).

$(2x+3)(2x-3)$, this is the factored form.

ex $16x^2 - 49$
$(4x + 7)(4x - 7)$
$16x^2 - 28 + 28 - 49$

You can check by simplifying.

$(2x+3)(2x-3)$
$4x^2 - 6x + 6x - 9$
$4x^2 - 9$

Practice Example:

Factor:
$100 - x^2$

$(x - 10)(x + 10)$

$x^2 + 10x - 10x + 100$

Factor:
$49x^2 + 36$

not factorable

Factor:
$16x^2 - 25$

$(4x - 5)(4x + 5)$

Factor:
$8x^2 - 50$

$2(4x^2 - 25)$

$2(2x + 5)(2x - 5)$

Factor:
$36x^2 - 81y^2$

$9(4x^2 - 9y^2)$

$9(2x + 3y)(2x - 3y)$

Factor:
$a^2 - b^2$

$(a - b)(a + b)$

$a^2 + ab - ab - b^2$

$a^2 - b^2$

How do you factor difference of two squares?

1) [illegible] 2) [illegible] 3) [illegible]
[illegible] form.

Factoring Perfect Square Trinomial

Example: Factor- $x^2 + 10x + 25$

$x^2 + 10x + 25$ is a perfect square trinomial because:

The first term: x^2 is the square of x — $x \cdot x = x^2$
The last term: 25 is the square of 5: — $5^2 = 25$
The second term: 10x is twice the product of x and 5: — 2(5)(x) = 10x

A trinomial is a perfect square trinomial if:

$$x^2 + 10x + 25$$

$$\sqrt{x^2} + 10x + \sqrt{25}$$

$$x \qquad\qquad 5$$

$$+2(5x)$$

$$x \quad + \quad 5$$

$$(x+5)^2$$

The factored form of: $x^2 + 10x + 25$ is: $(x+5)(x+5) = (x+5)^2$

Example: Factor- $9x^2 - 42x + 49$

$9x^2 - 42x + 49$ is a perfect square trinomial because:

The first term: $9x^2$ is the square of 3x — $3x \cdot 3x = 9x^2$
The last term: 49 is the square of 7: — $7^2 = 49$
The second term: -42x is - twice the product of 3x and 7: — -2(3x)(7) = -42x

A trinomial is a perfect square trinomial if:

$$9x^2 - 42x + 49$$

$$\sqrt{9x^2} - 42x + \sqrt{49}$$

$$3x \qquad\qquad 7$$

$$-2(3x)(7)$$

$$3x \quad - \quad 7$$

$$(3x-7)^2$$

The factored form of: $9x^2 - 42x + 49$ is: $(3x-7)(3x-7) = (3x-7)^2$

Practice Example:

Factor:
x^2+6x+9

x + $2(3x)$ 3

x + 3

$(x+3)(x+3)$

$(x+3)^2$

Factor:
$x^2-12x-36$

Factor:
$25x^2-30x+9$

$\sqrt{25x^2} - 30x + \sqrt{9}$

$5x - 2(x)3$

$5x - 3$

$(5x-3)^2$

$(5x-3)(5x-3)$

Factor:
$9x^2-30xy+25y^2$

$3x - 2(x)5$

$(3x-5)^2$

$(3x-5)(3x-5)$

5.4: Factoring Difference of Two Squares and Factor Perfect Square Trinomials Practice Problems

Factor the following:

1. $x^2 - 16$

   $(x-4)(x+4)$

2. $25 - x^2$

3. $9x^2 - 49$

4. $x^2 + 4$

   Prime

5. $-x^2 + 100$

6. $2x^2 - 8$

   $2(x^2-4)(\quad)$

   $2(x-2)(x+2)$

7. $4x^2 - 36$

   $4(x-3)(x+3)$

8. $c^2 - d^2$

   $(c+d)(c-d)$

9. $121x^2 - 81y^4$

   $11x$

5.4: Factoring Difference of Two Squares and Factor Perfect Square Trinomials Practice Problems Continue

Factor the following:

10. $x^2+18x+81$

11. $x^2+20x+100$

$x \quad +2 \quad +10$

$(x+10)^2$

12. $4x^2+28x+49$

$2x \quad +2 \quad +7$

$(2x+7)^2$

13. $x^2-10x+25$

$(x-5)(x-5)$

14. $x^2-8x+16$

$x \qquad 4$

15. $9x^2-12x+4$

$(3x-2)$

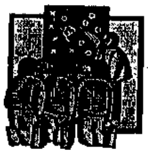
16. $25x^2+60x+36$

17. $49x^2-54x+16$

18. $9x^2-66xy+121y^2$

$3x \quad 2 \cdot 3 \cdot 11y \quad 11y$

$(3x-11y)^2$



5.5: Factoring Using Multiple Methods

If sometimes you need to factor using multiple methods, what process should you follow?

Steps for Factoring Polynomial

Check each method in this order:

1) Pull out the GCF
   Ex. $25x^3y^3 + 15x^5y^6 - 5x^2y^7$
   $5x^2y^3(5x - 3x^3y^3 - y^4)$

2) Grouping (Hint: 4 Terms or terms that can be paired)
   Ex. $2ys - 2yf + bs - bf$
   $(s - f)(2y + b)$

4) Trinomial where leading coefficient is 1
   Ex. $x^2 + 7x + 12$
   $(x + 3)(x + 4)$

5) Trinomial where the leading coefficient is not 1
   Ex. $12x^2 - 23x + 5$
   $(3x - 5)(4x - 1)$

6) Difference of Squares
   Ex. $4x^2 - 9$
   $(2x + 3)(2x - 3)$

If sometimes you need to factor using multiple methods, what process should you follow?

I pull out GCF, Grouping, Trinomial, Trinomial leader ... difference of Squares

Practice Examples:

Factor:

$8x^2 - 98$

$2(4x^2 - 49)$

Factor:

$-4x^5y^2 - 12x^4y^3 - 9x^3y^4$

Factor:

$-4x^2 + 32x + 4x^3$

$14x(x^2 - x + 8)$

Factor:

$-24xy + 42xy^2 + 12y^3x$

$6xy(2y^2 + 7y - 4)$

$2y^2 \quad -1y \quad 8y \quad -4$

$2y^2 - 1y \qquad 8y - 4$

$y(2y-1)$

5.5: Factoring Using Multiple Methods Practice Problems

1. Factor:

$$7x^2 - 28$$

$7(x^2-4)$

$7(x-2)(x+2)$

2. Factor:

$$2x^3 - 16x^2 + 24x$$

$2x(x^2-8x+12)$

3. Factor:

$$x^2(a+b) - 25(a+b)$$

$(a+b)(x^2-25)$

$(a+b)(x-5)(x+5)$

4. Factor:

$$4x^3 + 12x^2 - 9x - 27$$

$4x^3+12x^2 \quad -9x-27$

$4x^2(x+3) \quad -9(x+3)$

$4x^2-9 \quad (x+3)$

5. Factor:

$$-6x - 5x^2 + 6x^3$$

$-x(6+5x-6x^2)$

6. Factor:

$$a^2bc^2 - 10a^2bc + 24a^2b$$

$a^2b(c^2-10c+24)$ $\quad -4$

$a^2b(c-4)(c-6)$ $\quad -6$

5.6: Solve Quadratic Equations by Factoring

How do you solve quadratic equations by factoring?

Example: Solve: $2x^2 + 5x - 3 = 0$ by Factoring

In this method, MAKE SURE that the quadratic equation is equal to 0

Step 1: Factor the quadratic equation

$$2x^2 + 5x - 3 = 0$$
$$(2x-1)(x+3) = 0$$

Step 2: Set each factor equal to 0

$$(2x-1)(x+3) = 0$$
$$2x - 1 = 0 \qquad x + 3 = 0$$

Step 3: Solve each equation

$$\begin{array}{lcl} 2x - 1 = 0 & & \\ \quad +1 \quad +1 & & x + 3 = 0 \\ 2x \quad = 1 & & \quad -3 \quad -3 \\ \div 2 \quad \div 2 & & x \quad = -3 \\ x = 0.5 & & \end{array}$$

Step 4: Check your answers

Answers are <u>x = 0.5</u> and <u>x = -3</u>

Substitute each answer individually into the original equation:

$$2x^2 + 5x - 3 = 0 \qquad\qquad 2x^2 + 5x - 3 = 0$$
$$2(0.5)^2 + 5(0.5) - 3 = 0 \qquad\qquad 2(-3)^2 + 5(-3) - 3 = 0$$
$$0 = 0 \qquad\qquad 0 = 0$$

Example: Solve the quadratic equation: $x^2 - 4x - 12 = 0$

Step 1: Set the quadratic equation equal to zero

Step 2: Factor the quadratic equation

$(x-6)(x+2) = 0$

Step 3: Set each factor equal to zero

$x - 6 = 0$ $\qquad$ $x + 2 = 0$

Step 4: Solve each factor

$$\begin{array}{ll} x-6=0 & x+2=0 \\ \quad +6 \quad +6 & \quad -2 \quad -2 \\ x=6 & x=-2 \end{array}$$

Step 5: Check the solutions

Algebraically

$$\begin{array}{ll} x^2-4x-12=0 & x^2-4x-12=0 \\ (6)^2-4(6)-12=0 & (-2)^2-4(-2)-12=0 \\ 36-24-12=0 & 4+8-12=0 \\ 0=0 & 0=0 \end{array}$$

Practice Example: Solve: $6x^2 - 7x + 2 = 0$ by Factoring

$6x^2 - 7x + 2$ $\quad$ +12 mult

$\underline{6x^2}$ $\underline{-3x}$ $-4x$ $\underline{+2}$

$6x^2 - 3x$ $\qquad$ $-4x + 2$

$3x(2x-1)$ $\qquad$ $-2\ (2x-1)$

$2x - 1 = 0$ $\qquad$ $3x - 2 = 0$

$+1 \quad +1$

$2x = 1$

$\overline{2} \quad \overline{2}$

$x = \frac{1}{2}$

EXAMPLE: Solve the quadratic equation: $3x(x+1) = 2x + 2$

Step 1: Set the quadratic equation equal to zero

distribute $3x^2 + 3x = 2x + 2$

$-2x \quad -2x$

$3x^2 + x = 2$

$-2 \quad -2$

$3x^2 + x - 2 = 0$

Step 2: Factor the quadratic equation $3x^2 + x - 2 = 0$ -6 mult 1 add

$3x^2 + 3x - 2x - 2$

$3x^2 + 3x \qquad -2x - 2$

$3x(x+1) \qquad -2(x+1)$

$(x+1)(3x-2)$

Step 3: Set each factor equal to zero

$x + 1 = 0 \qquad 3x - 2 = 0$

Step 4: Solve each factor

$x + 1 = 0$

$-1 \quad -1$

$\boxed{x = -1}$

$3x - 2 = 0$

$+2 \quad +2$

$\frac{3x}{3} = \frac{2}{3}$

$\boxed{x = \frac{2}{3}}$

Step 5: Check the solutions

$3(-1)^2 + (-1) - 2 = 0$

$3\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right) - 2 = 0$

How do you solve quadratic equations by factoring?

by putting each equation to zero and [illegible]

5.6: Solve Quadratic Equations by Factoring Practice Problems

Solve:

1. $(x+2)(x-3)=0$

2. $x(x+6)(2x-1)=0$

3. $x^2-3x=0$

   $x(x-3)=0$

4. $5x^2+10x=0$

   $5x(x+2)=0$

5. $4x(x+7)-3(x+7)=0$

6. $8x^2+12x+10x+15=0$

5.6: Solve Quadratic Equations by Factoring Practice Problems Continue

Solve:

7. $x^2 - 11x + 30 = 0$

8. $x^2 - 100 = 0$

9. $12x^2 - 17x - 5 = 0$

10. $x^2 - 4x - 30 = 2$

$x^2 - 4x - 32 = 0$

$(x-8)(x+4) = 0$

$x - 8 = 0$

$+8 \quad +8$

$x = 8$

11. $-10 + 6x^2 = 11x$

12. $14x^3 + 19x^2 + 5x = 8x$

$-8x \quad -8x$

$14x^3 + 19x^2 - 3x = 0$

$x(14x^2 + 19x - 3) = 0$

$14x^2 - 2x + 21x - 3$

$2x(7x-1)$

$x = \frac{1}{7}$

5.7: Simplify Rational Expressions

How do you simplify rational expressions?

Example: **Simplify-** $\frac{10}{35}$

Solution:

Step 1: Factor the numerator and denominator

$$\frac{10}{35} = \frac{2 \cdot 5}{7 \cdot 5}$$

Step 2: Simplify

$$\frac{2 \cdot \cancel{5}}{7 \cdot \cancel{5}}$$

Step 3: **Final Answer**

$$\frac{2}{7}$$

Practice Examples:

Simplify- $\frac{45}{63}$ $\frac{\cancel{3} \cdot \cancel{3} \cdot 5}{\cancel{3} \cdot \cancel{3} \cdot 7}$ $\frac{5}{7}$

Simplify- $\frac{60}{140}$ $\frac{\cancel{2} \cdot \cancel{2} \cdot 3 \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot 7}$ $\frac{3}{7}$

Simplify- $\frac{25x^4y}{10x^2y^5}$ $\frac{5 \cdot 5x^4y}{2 \cdot \cancel{5}\, x^2y^5}$

$5x^2y^4$

Simplify- $\frac{14a^7b^4}{20a^3b^5}$ $\frac{\cancel{2} \cdot 7a^7b^4}{\cancel{2} \cdot 2 \cdot 5\, a^3 b^5}$

$\frac{7a^4}{10b}$

Example: **Simplify-** $\frac{3x^2-8x-3}{x^2-9}$

$$\frac{3x^2-8x-3}{x^2-9} \qquad \frac{(3x+1)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} \qquad \frac{3x+1}{x+3}$$

Solution:

<u>Step 1</u>: Factor the numerator and denominator

$$\frac{3x^2-8x-3}{x^2-9}=\frac{(3x+1)(x-3)}{(x+3)(x-3)}$$

<u>Step 2</u>: Simplify

$$\frac{(3x+1)(x-3)}{(x+3)(x-3)}$$

<u>Step 3</u>: **<u>Final Answer</u>**

$$\frac{3x+1}{x+3}$$

Practice Examples:

Simplify- $\frac{2x-14}{x^2-49}$ $\quad \frac{2\cancel{(x-7)}}{(x+7)\cancel{(x-7)}}$ $\quad \frac{2}{x+7}$

Simplify- $\frac{x^2+3x+2}{x^2+x-2}$ $\quad \frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-1)}$ $\quad \frac{x+1}{x-1}$

Practice Examples:

Simplify- $\frac{x^2-4}{x^2-2x-8}$ $\frac{\cancel{(x+2)}(x-2)}{(x-4)\cancel{(x+2)}}$

$\frac{x-2}{x-4}$

Simplify- $\frac{x^2+3x+2}{x^2+x-2}$ $\frac{\cancel{(x+2)}(x+1)}{(x-1)\cancel{(x+2)}}$

$\frac{x+1}{x-1}$

Simplify- $\frac{6x-30}{5-x}$

$\frac{6x-30}{-x+5} = \frac{6(x-5)}{-1\cancel{(x-5)}}$

$\frac{6}{-1} = \boxed{-6}$

Simplify- $\frac{6x^2-13x+6}{3x^2+x-2}$ $\frac{(2x-3)\cancel{(3x-2)}}{\cancel{(3x-2)}(x+1)}$

$\boxed{\frac{2x-3}{x+1}}$

$6x^2-13x+6$ 36 -6

$6x^2 - 9x - 4x + 6$ $3x^2+x-2$

$6x^2-9x$ $-4x+6$ $3x^2+3x-2x-2$

$3x(2x-3)$ $-2(2x-3)$, $3x(x+1)$ $-2(x+1)$

$(2x-3)(3x-2)$ $(3x+2)(x+1)$

How do you simplify rational expressions?

by breaking it down in terms

5.7: Simplify Rational Expressions Practice Problems

Simplify:

1. $\dfrac{7}{77}$

2. $\dfrac{50}{75}$

3. $\dfrac{21x^3y^2}{6xy^4}$ $\quad \dfrac{7x^2}{2y^2}$

4. $\dfrac{36x^5y^3}{72x^4y^3}$

5. $\dfrac{x^2-3x}{x-3}$

6. $\dfrac{5x}{25x^2-30x}$

7. $\dfrac{x^2+2x}{x^2+3x+2}$

8. $\dfrac{x^2+3x+xy+3y}{x^2+5x+6}$

5.7: Simplify Rational Expressions Practice Problems Continue

Simplify:

9. $\dfrac{x+3}{x^2+x-6}$

10. $\dfrac{x^2-2x-15}{x^2-8x+15}$

$\dfrac{(x-5)(x+3)}{(x-3)\cancel{(x-5)}}$ $\dfrac{x+3}{x-3}$

11. $\dfrac{x^2-16}{x^2-8x+16}$

$\dfrac{(x-4)(x+4)}{(x-4)(x-4)}$

12. $\dfrac{2x^2+11x+5}{2x^2-50}$

13. $\dfrac{3x^2+13x+12}{x^2-4x-21}$

$\dfrac{3x+4}{x-1}$

14. $\dfrac{6x^2-7x-3}{8x^2-6x-9}$

$\dfrac{3x+1}{4x+3}$

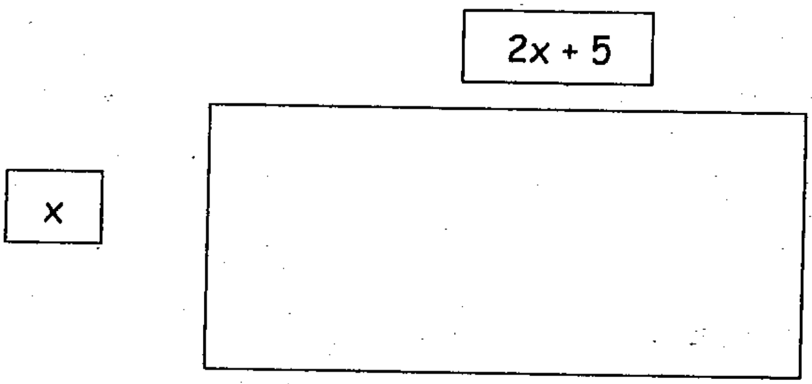
5.8: Factoring Applications

What are some examples where factoring is used in life?

Word Problem:

Find the area of a rectangle given that the length of the rectangle is 5 more than twice the width.

Solution:



To find the area of a rectangle use the formula for Area of a rectangle: Area = Length times Width or $A = L \cdot W$

In this case, the width is "x" and the length is "2x+5" so the area is:

$$A = L \cdot W$$
$$A = x(2x+5)$$
$$A = 2x^2 + 5x$$

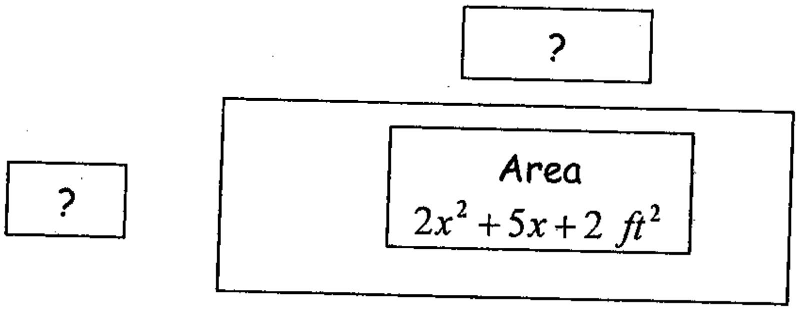
Factoring Application 1: Factoring can be used to find unknowns.

Example:

Find the length and width of a rectangle whose area is: $2x^2 + 5x + 2 \ ft^2$

(In this case, the length is the longer side and the width is the shorter side.)

Once the width and length algebraic expressions are found, find the value of the width and length when $x = 5$ ft. Then verify the area by substituting $x = 5$ ft into the area expression.



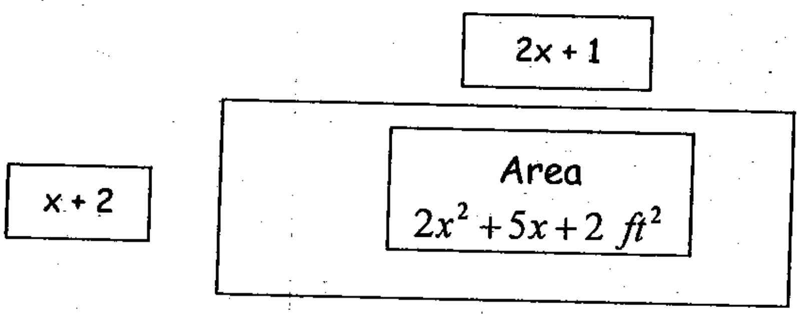
Solution:

To find the solution factor the area to find the length and width:

$$2x^2 + 5x + 2$$

$$(x+2)(2x+1)$$

The width is "x + 2" feet and the length "2x + 1" feet.



If $x = 5$ ft, then:

Width	Length	Area
$x+2$	$2x+1$	
$(5)+2$	$2(5)+1$	
$7 \ ft$	$11 \ ft$	$(7 \ ft)(11 \ ft) = 77 \ ft^2$

Area

$2x^2 + 5x + 2 \ ft^2$

$2(5)^2 + 5(5) + 2$

$50 + 25 + 2$

$77 \ ft^2$

Practice Example:

Find the length and width of a rectangle garden whose area is: $x^2 - 25 \quad ft^2$
(In this case the length is the longer side and the width is the shorter side.)
Once the width and length are found, find the value of the width and length when x = 10 ft. Then verify the area by substituting $\boxed{x = 10}$ ft into the area.

?= $x+5$ $\quad 10+5 = \boxed{15}$

?= $x-5$

Area
$x^2 - 25 \; ft^2$

$10-5$
$\boxed{5}$

$(x+5)(x-5)$

$15(5)$

$(10)^2 - 25$
$100 - 25$
$75 ft^2$

Practice Example:

Find the length and width of a square play area whose area is: $4x^2 + 12x + 9 \quad ft^2$
(In this case the length is the longer side and the width is the shorter side.)
Once the width and length are found, find the value of the width and length when x = 6 ft. Then verify the area by substituting $\boxed{x = 6}$ ft into the area.

?= $2x+3$ $\quad 2(6)+3$
15

?= $2x+3$
$2(6)+3$
15

Area
$4x^2 + 12x + 9 \; ft^2$

$4x^2 + 6x \quad 6x + 9$

$2x(2x+3) \quad 3(2x+3)$

$(2x+3)^2$

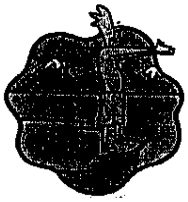
Area
$15(15) = 225 ft^2$

Check
$4(6)$

$2x \quad \boxed{12} \quad 3$

$(2x+3)^2$

Factoring Application 2: Solving real life equations



The equation of a person who dives off a 48 ft. cliff into a river with an initial velocity of 32 ft/sec is:

$\boxed{h = -16t^2 + 32t + 48}$ where "t" is time in seconds and "h" is height in feet.

When will the diver hit the water (when height = 0)?

Solution:

To find the solution:

1) Substitute 0 in for the height (h)

$$h = -16t^2 + 32t + 48$$
$$0 = -16t^2 + 32t + 48$$
$$-16t^2 + 32t + 48 = 0$$

2) Factor the trinomial by first factoring out the -16:

$$-16t^2 + 32t + 48 = 0$$
$$-16(t^2 - 2t - 3) = 0$$

3) Continue factoring by factoring the trinomial:

$$-16(t^2 - 2t - 3) = 0$$
$$-16(t-3)(t+1) = 0$$

4) Set each term with a variable equal to the right side of 0 and solve for t:

$$t - 3 = 0 \qquad t + 1 = 0$$
$$+3 \;\; +3 \qquad -1 \;\; -1$$
$$t = 3 \qquad t = -1$$

5) The solutions are 3 seconds and -1 seconds. Since -1 seconds is not realistic, we do not include that solution. The diver will hit the water is 3 seconds.

$2x-6$

$-(x-3)$

Practice Example:

The equation of a person who dives off a 64 ft. cliff into a river with an initial velocity of 0 ft/sec is:

$h = -16t^2 + 64$, where "t" is time in seconds and "h" is height in feet.

When will the diver hit the water (when height = 0)?

$-16t^2 + 64 = 0$

$-16(t^2 - 4) = 0$

$16(t+2)(t-2) = 0$

$t+2=0 \quad t-2=0$

$-2 \quad -2 \qquad +2 \quad +2$

$\boxed{\cancel{t=-2}} \quad \boxed{t=2}$

2 sec.

Practice Example:

The equation of the path of a ball thrown in the air at a speed of 96 ft/sec is:

$h = -16t^2 + 96t$, where "t" is time (seconds) and "h" is height in feet.

When will the ball hit the ground (when height = 0)?

$-16t^2 + 96t = 0$

$-16t(t-6) = 0$

$-16t = 0 \qquad t-6=0$

$\frac{-16t}{-16} = \frac{0}{-16} \qquad +6 \quad +6$

$\boxed{t=0} \qquad t=6$

6 second

What are some examples where factoring is used in life?

Solving for Areas equation in life

5.8: Factoring Applications Practice Problems

Solve:

1. Find the length and width of a rectangular room whose area is: $16x^2 - 25 \quad ft^2$
Draw a picture of the situation.
(In this case the length is the longer side and the width is the shorter side.)
Once the width and length are found, find the value of the width and length when x = 10 ft. Then verify the area by substituting x = 10 ft into the area.

$16x^2 - 25$

2. Find the length and width of a square room whose area is: $4x^2 + 20x + 25 \quad ft^2$
Draw a picture of the situation.
(In this case the length is the longer side and the width is the shorter side.)
Once the width and length are found, find the value of the width and length when x = 4 ft. Then verify the area by substituting x = 4 ft into the area.

5.8: Factoring Applications Practice Problems Continue

Solve:

3. The equation of a person who dives off a 96 ft. cliff into a river with an initial velocity of 16 ft/sec is:
$h = -16t^2 + 16t + 96$, where "t" is time in seconds and "h" is height in feet.
When will the diver hit the water (when height = 0)?

4. The equation of the path of a ball thrown in the air at a speed of 16 ft/sec is:
$h = -16t^2 + 16t$, where "t" is time (seconds) and "h" is height in feet.
When will the ball hit the ground (when height = 0)?

3/1?/?

MAT0024C Beginning Algebra Name: [illegible]

Test 3 (Chapter 5) REVIEW

Questions from Chapter 5

Factor:

1. $\frac{10x^6}{2} - \frac{6x^8}{2}$

$\frac{5x^6}{}$ $\frac{3}{}$

$2x^6(5 - 3x^2)$

2. $24v^4w^4 + 4vw^2$

$4vw^2(6v^3w^2 + 1)$

3. $45x^6y^{55} + 10x^{18}y^{15} + 55x^{24}y^{35}$

$5x^6y^{?}(9y^{?} + 2x^{12} + 11x^{18}y^{2?})$

4. $2x^2 + 8x + 3x + 12$

$2x^2 + 8x$ $3x + 12$

$2x(x+4)$ $3(x+4)$

$(2x+3)(x+4)$

5. $7x^3 - 21x^2 + x - 3$

$7x^3 - 21x^2$ | $x - 3$

$7x^2(x-3)$ | $+1(x-3)$

$\boxed{(7x^2+1)(x-3)}$

6. $15y^2 - 10yz + 6y - 4z$

$15y^2 - 10yz$ $6y - 4z$

$5y(3y - 2z)$ $2(3y - 2z)$

$(5y + 2)(3y - 2z)$

7. $x^2 + 11x + 30$

$(x+5)(x+6)$

8. $x^2 - 8x + 16$

$(x-4)(x-4)$

9. $x^2 - x - 6$

$(x+2(x-3)$

10. $4c^2 + 12c + 9$ 36 mult

$\underline{4c^2}+\underline{6c}+\underline{6c}\ \underline{+9}$ 12 add

$4c^2+6c \qquad 6c+9$

$2c(2c+3) \qquad 3(2c+3)$

$(2c+3)^2$

$4c^2+12c+9$

$(2c+3)^2$

11. $9n^2 - 9n - 10$

$\underline{9n^2}\ \underline{6n}\ \underline{15n}\ \underline{-10}$

$9n^2+6n \qquad -15n-10$

$3n(3n+2) \qquad -5(3n+2)$

$(3n-5)(3n+2)$

12. $12x^2 + 19x + 5$

$\underline{12x^2}\ \underline{+4x}+\underline{15x}\ \underline{+5}$

$12x^2+4x \qquad 15x+5$

$4x(3x+1) \qquad 5(3x+1)$

$(4x+5)(3x+1)$

13. $16z^2 - 25$

$(4z-5)(4z+5)$

14. $2b^2 - 2c^2$

$2(b+c)(b-c)$

Solve:

15. $x^2 - 8x = 0$

$x(x-8)$

$\boxed{x=0}$

$x - 8 = 0$
$+8 \quad +8$

$\boxed{x=8}$

16. $x^2 - 4x - 5 = 0$

$(x+1)(x-5)$

$x+1=0 \qquad x-5=0$
$-1 \quad -1 \qquad +5 \quad +5$

$\boxed{x=-1} \qquad \boxed{x=5}$

Solve:

17. $3x^2 + 2x - 8 = 0$

$(x+2)(3x-4)$

$x+2=0 \qquad 3x-4=0$
$-2 \quad -2 \qquad +4 \quad +4$

$\boxed{x=-2} \qquad \frac{3x}{3} = \frac{4}{3}$

$\boxed{x=\frac{4}{3}}$

18. $4x^2 + 3x - 1 = 0$

-4 mul
+3 add

$4x^2 + 4x \quad 1x \quad -1$

$(4x-1)(x+1) =$

$4x-1=0 \qquad x+1=0$
$+1 \quad +1 \qquad -1 \quad -1$

$\frac{4x}{4} = \frac{1}{4} \qquad \boxed{x=-1}$

$\boxed{x=\frac{1}{4}}$

19. Simplify: $\frac{x^2 - x - 56}{x^2 - 49}$

$\frac{\cancel{(x+7)}(x-8)}{(x-7)\cancel{(x+7)}}$

$\boxed{\frac{x-8}{x-7}}$

$x^2 - 6x + 2$

20. Simplify: $\frac{x^2 - 4x - 12}{2x^2 - 15x + 18}$

$\frac{(x+2)\cancel{(x-6)}}{(2x-3)\cancel{(x-6)}}$

$\boxed{\frac{x+2}{2x-3}}$

21. Two machines can complete 8 tasks every 3 days. Let t represent the number of tasks these machines can complete in a 30-day month. Write a proportion to show this example.

$$\frac{\text{task: } t}{\text{days: } 30} = \frac{8}{3}$$

22. Simplify: $7x+8(x-3)$

$7x + 8x - 24$

$\boxed{15x - 24}$

23. Convert to scientific notation: 0.000023

2.3×10^{-5}

24. Solve: $2x - 10 < -8$

$+10 \quad +10$

$\frac{2x}{2} < \frac{2}{2}$

$x < 1$

25. Simplify: $(a^2b^4)^3(a^3b^4)$

$a^6b^{12}(a^3b^4)$

$1a^9b^{16}$

26. Simplify: $|8+(-14)|+9$

$|8-14|+9$

$|-6|+9$

$6+9=15$

27. Solve for t: $x=-8z+7t$

$\frac{8z+x}{7}=\frac{7t}{7}$

$\frac{8z+1x}{7}=t$

$\frac{8z}{7}+\frac{1}{7}x=t$

28. Find the y –intercept for: $-5x+8y=-8$

$-5(0)+8y=-8$

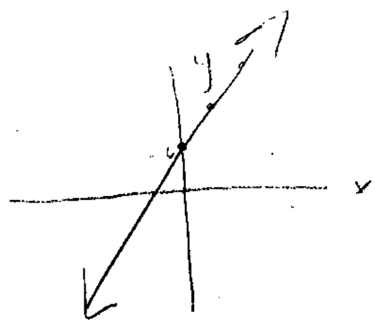
$0+\frac{8y}{8}=\frac{-8}{8}$

$y=-1$

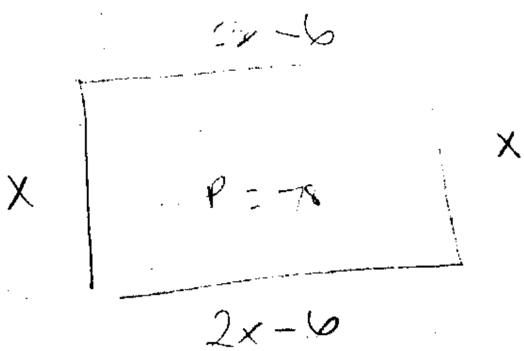
$(0,-1)$

29. Graph: $y = 3x+6$

$m=\frac{3}{1}$ $b=6$



30. The length of a rectangular pool is 6 less than twice the width. The perimeter of the pool is 78 feet. Find the length and width of the pool. Label each distance correctly.



$x=15$ width

$2x-6$

$2(15)-6=30-6$

24 length

$x+2x-6+x+2x-6=78$

$6x-12=78$

$+12$ $+12$

$6x=90$

$x=15$